Unit - IV.

method - I : Revised Simplex method.

1. Use the revised simplex method to solve the following LPP. Maximize $Z = 2x_1 + x_2$ Subject to the constraints.

$3x_1 + 4x_2 \le 6$.

$6x_1 + x_2 \le 3$ and $x_1, x_2 \ge 0$.

Soln:

Step - 1:

Reduce standard form of LPP.

$$Z = 2x_1 + x_2.$$

$$3x_1 + 4x_2 + s_1 = 6.$$

$$6x_1 + x_2 + s_2 = 3 \qquad , x_1, x_2, s_1, s_2 \ge 0.$$

Number of non-basic variables = no. of variables - no. of constraints.

$= 4 - 2$

$= 2$.

Let x_1 and x_2 are non basic and let $x_1 = x_2 = 0$.

$\therefore s_1 = 6$, $s_2 = 3$. basic feasible solution.

Reduce Z purely Interms of non basic variables.

Already Z interms of non. basic variables

Basic variables s_1 s_2

Non-basic variables x_1 x_2.

$Z - 2x_1 - x_2 = 0$.

$3x_1 + 4x_2 + s_1 = 6$.

$6x_1 + x_2 + s_2 = 3$.

Now Co. Efficient matrices

Z-Co-Efficient $P_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

x_1-Co-Efficient $P_1 = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$

x_2-Co-Efficient $P_2 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$

s_1-Co-Efficient $P_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

s_2-Co-Efficient $P_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

Table : I.

$B^{-1}_{3\times 3}$.

Pivot column.

	Z	s_1	s_2	Soln	y_i	θ	
Z	1	0	0	0	-2		
s_1	0	1	0	6	3	$\frac{6}{3} = 2$	
← s_2	0	0	1	3	$\boxed{6}$	$\frac{3}{6} = \frac{1}{2}$.	← Least +ve ratio.

Pivot element.

To find entering variable :

$= \text{least -ve of } (I^{st} \text{ row } B^{-1})$ (non basic variable matrix)

$$= \text{least -ve of } (1 \; 0 \; 0) \begin{array}{cc} x_1 & x_2 \\ \end{array} \begin{pmatrix} -2 & -1 \\ 3 & 4 \\ 6 & 1 \end{pmatrix}$$

$= \text{least -ve of } (-2 \quad -1)$ corresponding to x_1

$\therefore$ x_1 Entering variable.

To find y_i :

$y_i = B^{-1} P_i$, where P_i Co. Efficient matrix corresponding to the entering variable

(current)

$$y_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}.$$

$$y_i = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}.$$

$\therefore$ S_2 leaves variable.

Basic s_1 x_1.

non-basic s_2 x_2.

$s_1 \leftrightarrows s_2$, x_1 x_2

Table : II.

Pivot column.

$B^{-1}_{3\times3}$: Z	s_1	x_1	Soln	y_i	θ
Z 1	0	1/3	1	-2/3	
← s_1 0	1	-1/2	9/2	7/2	$\frac{9}{7} = 1$. ← ratio
New pivot row → x_1. 0	0	1/6	1/2	1/6	3

least +ve.

$\frac{9}{2} \times \frac{2}{7}$

$\frac{1}{2} \times 6$

$-\frac{1}{2} \times \frac{2}{7}$

$\frac{9}{2} \times \frac{2}{7}$

Z-Co.eff: $1 + (2 \cdot 0) = 1$; $0 + (2 \cdot 0) = 0$; $0 + (2 \cdot \frac{1}{6}) = \frac{1}{3}$; $0 + (2 \cdot \frac{1}{2}) = 1$.

s_1 Co.eff : $0 - (3 \cdot 0) = 0$; $1 - (3 \cdot 0) = 1$; $0 - (3 \cdot \frac{1}{6}) = -\frac{1}{2}$; $6 - (3 \cdot \frac{1}{2}) = 6 - \frac{3}{2} = \frac{9}{2}$

To find Entering variable.

= least -ve of (I^{st} row B^{-1}) (non basic variables matrix)

$$= \text{least -ve of } \begin{pmatrix} 1 & 0 & 1/3 \end{pmatrix} \begin{array}{cc} s_2 & x_2 \\ \end{array} \begin{pmatrix} 0 & -1 \\ 0 & 4 \\ 1 & 1 \end{pmatrix}$$

1×3 3×2

$$= \text{least -ve of } \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} \end{pmatrix}$$ corresponding to x_2.

$-1 + \frac{1}{3}$

To find y_i :

$y_i = B^{-1} P_i$ (current), where P_i co.efficient matrix corresponding to the entering variable.

$$= \begin{pmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1/6 \end{pmatrix} \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$$

3×3 3×1

$$y_i = \begin{pmatrix} -1+\frac{1}{3} \\ 4-\frac{1}{2} \\ \frac{1}{6} \end{pmatrix}$$

$$y_i = \begin{pmatrix} -\frac{2}{3} \\ \frac{7}{2} \\ \frac{1}{6} \end{pmatrix}.$$

$\therefore$ s_1 leaves variable.

Basic	x_2	x_1.
non-basic	s_2	s_1.

Table : III.

	Z	x_2	x_1	Sdn	y_i	θ
Z	1	4/21	5/21	13/7.		
New pivot row → x_2	0	2/7	-1/7.	9/7.		
x_1	0	-1/21	4/21	2/7.		

Z-co-eff : $1 + \left(\frac{2}{3} \cdot 0\right) = 1$; $0 + \left(\frac{2}{3} \cdot \frac{2}{7}\right) = \frac{4}{21}$; $\frac{1}{3} + \left(\frac{2}{3} \cdot \frac{-1}{7}\right) = \frac{7-2}{21} = \frac{5}{21}$.

$1 + \left(\frac{2}{\cancel{3}} \cdot \frac{\overset{3}{\cancel{9}}}{7}\right) = 1 + \frac{6}{7} = \frac{13}{7}$.

x_1-co-eff : $0 - \left(\frac{1}{6} \cdot 0\right) = 0$; $0 - \left(\frac{1}{\cancel{6}_3} \cdot \frac{\cancel{2}}{7}\right) = \frac{-1}{21}$; $\frac{1}{6} + \left(\frac{1}{6} \cdot \frac{1}{7}\right) = \frac{\overset{4}{\cancel{8}}}{\cancel{42}_{21}}$

$\frac{1}{2} - \left(\frac{1}{\cancel{6}_2} \cdot \frac{\overset{3}{\cancel{9}}}{7}\right) = \frac{4}{14} = \frac{2}{7}$.

To find entering variable :

$$= \text{least -ve of } \begin{pmatrix} 1 & \frac{4}{21} & \frac{5}{21} \end{pmatrix} \begin{array}{cc} s_2 & s_1 \\ \end{array} \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \text{least -ve of } \left(\frac{5}{21} \quad \frac{4}{21}\right).$$

$\therefore$ No Negative values.

$\therefore$ Last table is optimum table.

$\therefore$ optimal solution is

$$\max.\ z = \frac{13}{7}.$$

$$x_1 = \frac{2}{7},\ x_2 = \frac{9}{7}.$$

2. Use the revised simplex method to solve the following LPP. Maximize $Z = x_1+2x_2+3x_3$ subject to the constraints

$$x_1+2x_2+3x_3 \le 10.$$

$$x_1+x_2 \le 5. \quad ,\ x_1, x_2, x_3 \ge 0.$$

Soln: Reduce standard form of LPP.

$$\max.\ Z = x_1+2x_2+3x_3.$$

Subject to,

$$x_1+2x_2+3x_3+s_1=10.$$

$$x_1+x_2+s_2=5 \quad ,\ x_1, x_2, x_3, s_1, s_2 \ge 0.$$

Number of non-basic variables = no. of variables − no. of constraints

$= 5 - 2$

$= 3.$

Let x_1, x_2, x_3 are non-basic variables and let $x_1=x_2=x_3=0$.

$\therefore s_1=10,\ s_2=5$ basic feasible solution.

Reduce Z purely interms of non basic variables.

Already z interms of non-basic variables.

$$Z-x_1-2x_2-3x_3=0.$$

Basic variables s_1 s_2

non-basic variables x_1 x_2 x_3.

$$x_1+2x_2+3x_3+s_1=10$$

$$x_1+x_2+s_2=5.$$

Now Co-Efficient matrices,

Z-Co-Efficient $\quad$ x_1-Co-Efficient $\quad$ x_2-Co-Efficient $\quad$ x_3-Co-Efficient

$$P_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad P_1 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \quad P_2 = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad P_3 = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}.$$

s_1-Co-Efficient $\quad$ s_2-Co-Efficient

$$P_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad P_5 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Table: I. $\quad B^{-1}_{3\times3}$.

	Z	s_1	s_2	Soln	y_i (Pivot column)	θ
Z	1	0	0	0	-3	
$\leftarrow s_1$	0	1	0	10	$\boxed{3}$ Pivot Element	$\frac{10}{3} = 3.3$ $\leftarrow$ least +ve ratio
s_2	0	0	1	5	0	$\frac{5}{0} = \infty$

To find Entering variable:

= least -ve of (I^{st} row of B^{-1}) (non-basic variable matrix)

$$= \text{least -ve of } (1 \;\; 0 \;\; 0) \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} -1 & -2 & -3 \\ 1 & 2 & 3 \\ 1 & 1 & 0 \end{pmatrix} \end{array}.$$

$$= \text{least -ve of } (-1 \;\; -2 \;\; -3) \text{ Corresponding to } x_3.$$

$\therefore$ x_3 Entering variable.

To find y_i:

$y_i = \underset{\text{(current)}}{B^{-1}} P_i$, where P_i Co-Efficient matrix Corresponding to the Entering variable.

$$y_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}.$$

$$y_i = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}$$

$\therefore$ s_1 leaves variable.

Basic : x_3 s_2

non-basic : x_1 x_2 s_1

Table : II.

$$\overbrace{Z \quad x_3 \quad s_2}^{B^{-1}_{3\times3}}$$

	Z	x_3	s_2	Soln.	y_i	θ
Z	1	1	0	10		
New pivot row → x_3	0	$1/3$	0	$10/3$		
s_2.	0	0	1	5.		

Z-co-efficient: $1+(3\cdot0)=1$; $0+(3\cdot\frac{1}{3})=1$; $0+(3\cdot0)=0$; $0+(3\cdot\frac{10}{3})$

~~s_2-co-efficients:~~

To find Entering variable:

= least -ve of (I^{st} row B^{-1}) (non basic variables matrix)

$$= \text{least -ve of } (1 \quad 1 \quad 0) \begin{pmatrix} -1 & -2 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}_{1\times3}$$

= least -ve of $(0 \quad 0 \quad 1)$

$\therefore$ No negative values. Last table is optimum table

$\therefore$ optimal Solution is max. Z = 10.

$x_1 = 0$, $x_2 = 0$, $x_3 = \frac{10}{3}$.

2. Solve the following LPP maximize $Z = 3x_1 + 5x_2 + 2x_3$.

Subject to $x_1 + 2x_2 + 2x_3 \le 14$.

$2x_1 + 4x_2 + 3x_3 \le 23$.

and $0 \le x_1 \le 4$, $2 \le x_2 \le 5$, $0 \le x_3 \le 3$.

Sol:

Reduce lower bound as zero.

$2 \le x_2 \le 5$.

$2-2 \le x_2 - 2 \le 5-2$.

$0 \le x_2 - 2 \le 3$.

Let $x_2' = x_2 - 2$.

$0 \le x_2' \le 3$.

$\therefore \boxed{x_2 = x_2' + 2}$

Put $x_2 = x_2' + 2$.

$\text{Max. } Z = 3x_1 + 5(x_2' + 2) + 2x_3$.

Subject to, $x_1 + 2(x_2' + 2) + 2x_3 \le 14$.

$\Rightarrow x_1 + 2x_2' + 2x_3 \le 10. \rightarrow (1)$

$2x_1 + 4(x_2' + 2) + 3x_3 \le 23$.

$\Rightarrow 2x_1 + 4x_2' + 3x_3 \le 15 \rightarrow (2)$

$0 \le x_1 \le 4$, $0 \le x_2' \le 3$, $0 \le x_3 \le 3$.

Reduce standard form of LPP,

$x_1 + 2x_2' + 2x_3 + s_1 = 10$.

$2x_1 + 4x_2' + 3x_3 + s_2 = 15$, x_1, x_2', x_3, s_1

Number of non basic variables = no. of variables − no. of constraints

$= 5 - 2$

$= 3$.

Let x_1, x_2', x_3 are non basic and let $x_1 = x_2' = x_3 = 0$

$\therefore \; s_1 = 10, \; s_2 = 15$.

Basic : $s_1 \quad s_2$

$Z = 3x_1 + 5x_2' + 10 + 2x_3$. non-basic : $x_1 \quad x_2' \quad x_3$.

$Z - 3x_1 - 5x_2' - 2x_3 = 10$

Table: I. (pivot column: x_2')

	x_1	x_2'	x_3	s_1	s_2	Soln.	u_i
Z	-3	-5	-2	0	0	10	
s_1	1	2	2	1	0	10	∞
s_2	2	4	3	0	1	15	∞.

$\therefore x_2'$ entering variable.

To find leaves variable : Determine θ.

$\theta = \min\{\theta_1, \theta_2, u_2'\}$, where u_2' is upper bound of x_2' ($\because x_2'$ entering variable)

$\theta_1 = \min\left\{\frac{10}{2}, \frac{15}{4}\right\} = \min\left\{5, \frac{15}{4}\right\}$

$= \frac{15}{4}$ corresponding to s_2. ; $\theta_2 = \infty$ ($\because$ no negative co-efficient)

$\therefore \theta = \min\left\{\frac{15}{4}, \infty, 3\right\}$.

$\theta = 3$ corresponding to x_2'.

$\therefore x_2'$ leaves variable.

Here entering and leaves variable is same.

$\therefore$ Replace x_2' by x_2''. and multiply (-1) in the pivot column.

$$X_B^{new} = X_B^{old} - ((\text{element in the pivot column})(u_2'))$$

$$= 10+5\cdot 3 = 25 \;;\; 10-2\cdot 3 = 4 \;;\; 15-4\cdot 3 = 3$$

$\therefore \boxed{x_2'' = u_2' - x_2'}$

Table: II

pivot column

	x_1	x_2''	x_3	s_1	s_2	soln	u_i
Z	-3	5	-2	0	0	25	
s_1	1	-2	2	1	0	4	∞
leaves s_2	2	-4	3	0	1	3.	∞.

$\therefore$ x_1 entering variable.

To find leaves variable : Determine θ.

$\theta = \min\{\theta_1, \theta_2, u_1\}$. , where u_1 is upper bound of x_1.

$\theta_1 = \min\left\{\frac{4}{1}, \frac{3}{2}\right\} = \min\{4, \frac{3}{2}\}$.

$\theta_1 = \frac{3}{2}$ corresponding to s_2.

$\theta_2 = \infty$ ($\because$ no negative value)

$\therefore$ $\theta = \min\{\frac{3}{2}, \infty, 4\}$

$\theta = \frac{3}{2}$ corresponding to s_2.

$\therefore$ s_2 leaves variable.

Table : III

pivot column

	x_1	x_2''	x_3	s_1	s_2	Soln	u_i
Z	0	-1	5/2	0	3/2	59/2	
s_1	0	0	1/2	1	-1/2	5/2	∞.
← x_1	1	-2	3/2	0	1/2	3/2	4.

$\therefore x_2''$ entering variable.

To find leaves variable : Determine θ.

$\theta = \min\{\theta_1, \theta_2, u_2''\}$, where u_2'' is upper bound of x_2''.

$\theta_1 = \min\left\{\frac{5/2}{0}\right\} = \infty$ corresponding to s_1.

$\theta_2 = \min\left\{\frac{4-\frac{3}{2}}{-(-2)}\right\} = \min\left\{\frac{5/2}{2}\right\} = \frac{5}{4}$ corresponding to x_1.

$\therefore \theta = \min\{\infty, \frac{5}{4}, 3\}$.

$= \frac{5}{4}$ corresponding to x_1.

$\therefore x_1$ leaves variable.

Table : IV.

	x_1	x_2''	x_3	s_1	s_2	Soln	u_i
Z	-1/2	0	7/4	0	5/4	115/4	
s_1	0	0	1/2	1	-1/2	5/2	∞.
x_2''	$\frac{-1}{2}$	1	-3/4	0	-1/4	-3/4.	3

Z-co. eq : $0+(1\cdot\frac{-1}{2}) = \frac{-1}{2}$; $\frac{5}{2}-\frac{3}{4}=\frac{7}{4}$; $\frac{3}{2}-\frac{1}{4}=\frac{5}{4}$; $\frac{59}{2}-\frac{3}{4}=\frac{115}{4}$.

~~Sp...~~ pivot element is 0 then same row.

$\therefore x_1$ entering variable.

To find leaves variable : Determine θ.

$\theta = \min\{\theta_1, \theta_2, u_1\}$, where u_1 is upper bound of x_1

$\theta_1 = \min\left\{\frac{5/2}{0}\right\} = \infty$ Corresponding to s_1.

$\theta_2 = \min\left\{\frac{3+3/4}{-(-\frac{1}{2})}\right\} = \min\left\{\frac{15}{\cancel{4}_2}\left(\frac{\cancel{2}}{1}\right)\right\} = \frac{15}{2}$ Corresponding to x_2''.

$\therefore \theta = \min\{\infty, \frac{15}{2}, 4\}$.

$= 4$ corresponding to x_1.

$\therefore x_1$ leaves variable.

Here entering and leaves variables are same.

$\therefore$ Replace x_1 by x_1'. and multiply (-1) in the pivot column.

$x_B^{new} = x_B^{old} - [(\text{element in the pivot column}) \cdot (u_1)]$.

$= \frac{115}{4} + \left[\left(\frac{1}{\cancel{2}}\right) \cdot \left(\cancel{4}^2\right)\right] = \frac{123}{4}$; $\frac{5}{2} - (0 \cdot 4) = \frac{5}{2}$;

$-\frac{3}{4} + \left(\frac{1}{\cancel{2}} \cdot \cancel{4}^2\right) = \frac{5}{4}$.

$\therefore \boxed{x_1' = u_1 - x_1}$

Table: V :

	x_1'	x_2''	x_3	s_1	s_2	Soln	u_i
Z	1/2	0	7/4	0	5/4	123/4	
s_1	0	0	1/2	1	-1/2	5/2	∞
x_2''	1/2	1	-3/4	0	-1/4	5/4.	3.

Since all the Z-co-efficients are ≥ 0.

$\therefore \boxed{\text{Max. } Z = \frac{123}{4}}$

$\therefore x_1 = u_1 - x_1'$; since $x_1' = 0$.

$\Rightarrow x_1 = 4 - 0$.

$\therefore \boxed{x_1 = 4.}$

Since $x_2'' = \frac{5}{4} \Rightarrow x_2' = u_2' - x_2''$

$x_2' = 3 - \frac{5}{4} = \frac{7}{4}$

Also, $x_2 = x_2' + 2$.

$x_2 = \frac{7}{4} + 2$

$\therefore \boxed{x_2 = \frac{15}{4}.}$

$\therefore \boxed{x_3 = 0.}$